Rigid Body Rotation

Week 9, Lesson 2

- Rotational Work & Kinetic Energy
- Rotational Inertia

References/Reading Preparation:
Schaum's Outline Ch. 10
Principles of Physics by Beuche – Ch.8

Rotational Work and Kinetic Energy

It is easy to see that a rotating object has kinetic energy.

Recall that for *linear motion*: KE = $\frac{1}{2}mv^2$

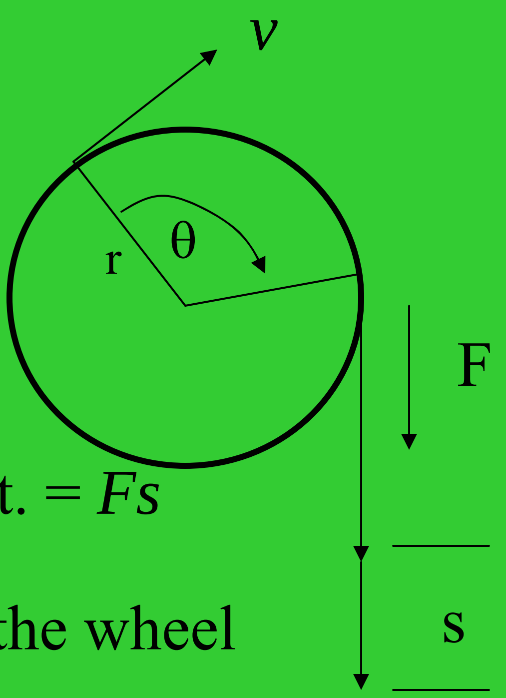
(we call this the ***translational kinetic energy – KE_t***)

When we consider that a rotating object is made up of many tiny bits of mass, each moving as the object turns, we can see that each tiny mass has a mass and a velocity.

Therefore, each mass, has kinetic energy of $\frac{1}{2}m_1v_1^2$

Consider this wheel with a string attached:

When a force F pulls on the string, the wheel begins to rotate.



The work done by the force as it pulls the string is:

Work done by F = Force x dist. = Fs

As the length (s) of string is unwound, the wheel turns through an angle θ.

Recall that $s = r\theta$. Therefore, work done by F = Fs = $Fr\theta$

The term *Fr (force x dist.)* is the torque, τ.

Therefore: $W = \tau\theta$

According to the work-energy theorem, the work done on the wheel by the applied torque *must appear* as Kinetic Energy, KE.

We call the Kinetic Energy resident in a rotating object the Kinetic Energy of Rotation, and is designated as:

$$KE_r$$

If we look at a wheel made up of many tiny masses, undergoing a velocity, then each mass has KE. The KE of the wheel is then the sum of the kinetic energies of all of the tiny masses.

KE of the wheel = $\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 + \frac{1}{2}m_3v_3^2 + \ldots + \frac{1}{2}m_nv_n^2$

Each tiny mass travels around a circle with radius r_n.

For a mass m_1, with velocity v_1, its angular velocity is related to the tangential velocity by $v_1 = \omega r_1$

Therefore, $\frac{1}{2}\, m_1v_1^2 = \frac{1}{2}\, m_1\omega^2r_1^2$

KE of the wheel = $\frac{1}{2}m_1\omega^2r_1^2 + \frac{1}{2}m_2\omega^2r_2^2 + \ldots + \frac{1}{2}m_n\omega^2r_n^2$

KE of the wheel = $\frac{1}{2} m_1 \omega^2 r_1^2 + \frac{1}{2} m_2 \omega^2 r_2^2 + \ldots + \frac{1}{2} m_n \omega^2 r_n^2$

Taking out common factors,

KE of the wheel = $\frac{1}{2}\omega^2 (m_1 r_1^2 + m_2 r_2^2 + \ldots m_n r_n^2)$

The term in the brackets is called the **moment of inertia (I)** of the rotating object.

Therefore, $KE_r = \frac{1}{2} I\omega^2$ (rotational kinetic energy)

If we apply a torque (τ) to a rotating wheel;

$\tau = I\alpha$ Where α is in rad/s.

Rotational Inertia

We know that rotating objects have *inertia*.

When we turn off a fan, the blade coasts for some time as the friction forces of the air and the axle bearings slowly cause it to stop.

The moment of inertia, I, of the fan blade measures its rotational inertia.

We can understand this as follows

In linear motion, the inertia of an object is represented by the object's mass.

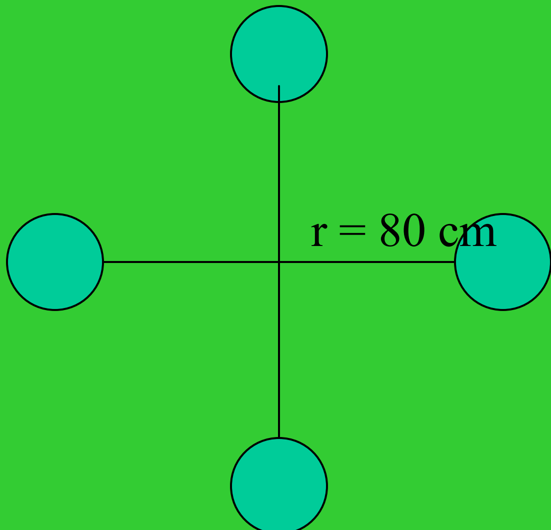
From $F = ma$, we have: $m = F/a$

If we fix 'a' at an acceleration of 1 m/s^2, then this equation shows us that the mass tells us how large a force is required to produce the given acceleration.

larger mass, then, larger force

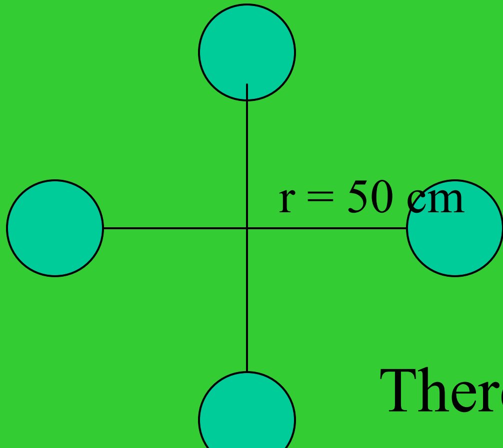
smaller mass, then, smaller force

Let's look at these two wheels, each having equal masses of 3 kg.



$$I = m_1r_1^{\ 2} + m_2r_2^{\ 2} + \ldots = \Sigma m_i r_i^{\ 2}$$

$$I_A = 3(0.8)^2 + 3(0.8)^2 + 3(0.8)^2 + 3(0.8)^2$$
$$= 7.68 \text{ kg·m}^2$$



$$I_B = 3(0.5)^2 + 3(0.5)^2 + 3(0.5)^2 + 3(0.5)^2$$
$$= 3.00 \text{ kg·m}^2$$

Note that (B) has a smaller moment of inertia Their masses are the same. The I's differ because the masses are further from the axis.

Therefore a greater torque is needed in A than in B.

The moment of inertia for any object s calculated by dividing the object into tiny masses and using using calculus.

The moments of inertia (about an axis through the centre of mass) of several uniform objects are shown in your text.

In all cases, I is the product of the object's mass and the square of a length.

Generally we can write the equation in the form $I = Mk^2$

Where M is the total mass of the object, and
k is the *radius f gyration*, and is the distance a point mass M must be from the axis if the point mass is to have the same I as the object.

Summary

1) An object of mass M possesses rotational inertia, where,

$$I = Mk^2$$

2) A rotating object has rotational kinetic energy, where,

$$\mathrm{KE}_r = \tfrac{1}{2} I\omega^2$$

3) A torque (τ) applied to an object that is free to rotate gives the object an angular acceleration, where,

$$\tau = I\alpha$$

4) The work done by a torque, τ, when it acts through an angle θ is $\tau\theta$.